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Phase diagram for Nflation

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ABSTRACT

Recently, it was showed that there is a large N phase transition in Nflation, in which when the number of fields is large enough, the slow roll inflation phase will disappear. In this brief report, we illustrate the phase diagram for Nflation, and discuss the entropy bound and some relevant results. It is found that near the critical point the number of fields saturates dS entropy.

1. Introduction

Recently, Dimopoulos et al. [1] showed that the many massive axion fields predicted by string vacuum can be combined and lead to a radiatively stable inflation, called Nflation, which is an interesting implement of assisted inflation mechanism proposed by Liddle et al. [2], see also Refs. [3,4] for many studies, and a feasible embedding of inflation in string theory. Then Easther and McAllister found [5] that for the mass distribution following Marčenko–Pastur law, the spectral index of scalar perturbation is always redder than that of its corresponding single field. However, this result is actually valid for any mass contribution and initial condition of fields, as has been shown in [6,7] numerically and in [8] analytically. In addition, it was found for Nflation that the ratio of tensor to scalar is always same as in the single field case [9] and the non-Gaussianity is small [10,11], see also Ref. [12] for relevant studies.

In inflation, when the value of field increases up to some value, the quantum fluctuation of field will be expected to overwhelm its classical evolution. In this case, the inflaton field will undergo a kind of random walk, which will lead to the production of many new regions with different energy densities. This was called as eternal inflation [13,14]. In principle, it was thought that dependent on the value of field, there are generally three different phases in single field inflation, i.e. eternal inflation phase, slow roll inflation phase and fast roll phase, which should be also valid for Nflation.

However, recently, it was found [15] that when the number of fields is large enough, the slow roll inflation phase will disappear, which means there exists a large N transition for Nflation. The reason is, though the end value of slow roll inflation decreases with the increase of number N of fields, the value separating the slow roll inflation phase and the eternal inflation phase, hereafter called as the eternal inflation boundary for convenience, decreases more rapidly, thus they will cross inevitably at some value of N, after this the slow roll inflation phase will go out of sight. This result means there is a bound for the number of fields driving the slow roll Nflation. This is also consistent with recent arguments from black hole physics [16,17], in which there exists a gravitational cutoff, whose value equals to our bound, beyond which the quantum gravity effect will become important, see also Refs. [18,19] for some similar bounds.

In single field inflation, when the inflaton field is in its eternal inflation boundary the primordial density perturbation $\delta\rho/\rho \sim 1$, thus it will be hardly possible for us to receive the information from the eternal inflation phase, since in that time we will be swallowed by black hole [20]. This result may be actually assured by a relation between the entropy and the total efolding number [21], in which when $\delta\rho/\rho \sim 1$, the entropy in unit of efolding number is less than one, which means we cannot obtain any information. Thus it is significant to examine how above results change for Nflation, especially what occurs around its phase transition point. It can be expected that there maybe more general and interesting results. In this Letter, we will firstly illustrate the phase diagram for Nflation, and then give relevant discussions.

2. Phase diagram for Nflation

In the Nflation model, the inflation is driven by many massive fields. For simplicity, we assume that the masses of all fields are equal, i.e. $m_i = m$, and also $\phi_i = \phi$, which will also be implemented in next section. Following Ref. [15], the end value of slow roll inflation phase and the eternal inflation boundary with respect to N are given by

$$\phi \simeq \frac{M_P}{\sqrt{N}}, \tag{1}$$

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$$\phi \simeq \frac{1}{N^{3/4}}\sqrt{\frac{M_p^3}{m}}, \tag{2}$$

respectively. It can be noticed that the end value goes along with $\frac{1}{\sqrt{N}}$, it decreases slower than the eternal inflation boundary with N, since the latter changes with $\frac{1}{N^{3/4}}$. Thus when we plot the lines of the end value and the eternal inflation boundary moving with respect to N, respectively, there must be a point where these two lines cross, see Fig. 1. This crossing point is

$$N \simeq \frac{M_p^2}{m^2}, \tag{3}$$

beyond which the slow roll inflation phase will disappear. Thus here we call this point as the critical point. It seems be expected that after the critical point is got across, the line denoting the eternal inflation boundary will not extend downwards any more, the line left is that denoting the end value, which still obeys Eq. (1), see the dashed line of Fig. 1. The reason is the calculation of the eternal inflation boundary is based on the slow roll approximation, while below the end value the slow rolling of field is actually replaced by the fast rolling, in this case the quantum fluctuation is actually suppressed, thus it is hardly possible that the quantum fluctuation of field will overwhelm its classical evolution. However, the case maybe not so simple. In next section, we will see there is an entropy bound for the number of fields, and at the critical point this bound is saturated. This means that beyond the critical point our above semiclassical arguments cannot be applied. Thus in this sense in principle what is the diagram beyond the critical point remains open.

The value of fields at critical point can be obtained by substituting Eq. (3) into any one of Eqs. (1) and (2), which is $\phi \simeq m$. This indicates that if initially $\phi < m$, no matter what N is, the slow roll inflation will not occur. The existence of slow roll inflation is important for solving the problems of standard cosmology and generating the primordial perturbation seeding large scale structures. In the phase diagram Fig. 1, we can see that the slow roll inflation phase is in a limited region, which means in order to make Nflation responsible for our observable universe, the relevant parameters must be placed suitably.

We assume that all mass are equal only for simplicity. For the case that not all mass are equal, the result is also similar, as has been shown in Ref. [15], in which the mass distribution following Marčenko–Pastur law [5] is taken for calculations. Thus the phase diagram is still Fig. 1, the only slight difference is replacing m with the average mass $\bar{m}$. It should be noted that here in the phase diagram the number N of fields dose not include massless scalar fields. The reason is when the masses of fields are negligible, they will not affect the motion of massive fields dominating the evolution of universe, while the perturbations used to calculate the quantum jump of fields are those along the trajectory of fields space, since the massless fields only provide the entropy perturbations orthogonal to the trajectory, which thus are not considered in the calculations deducing Eqs. (1) and (2). Thus if there are some nearly massless fields and some massive fields with nearly same order, it should be that there is a bound $N \lesssim M_p^2/\bar{m}^2$, in which only massive fields are included in the definition of $\bar{m}$ and N.

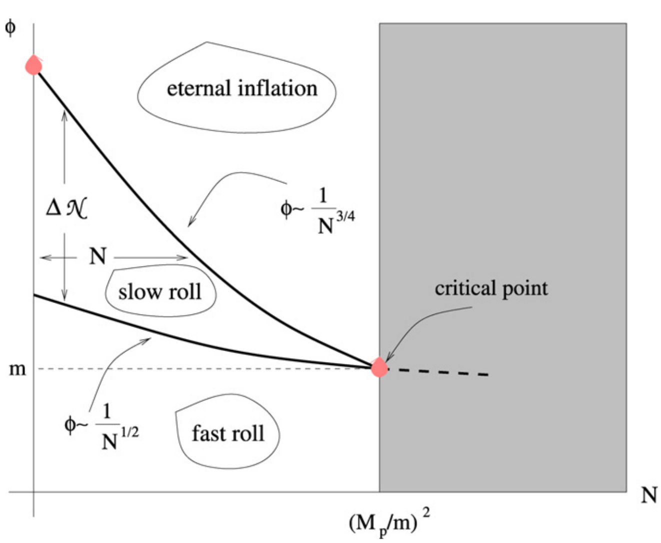

Fig. 1. The ϕ–N phase diagram for Nflation. The upper solid line is the eternal inflation boundary and the lower solid line is the end value of the slow roll inflation. These two lines split the region into three phases, i.e. eternal inflation phase, slow roll inflation phase and fast roll phase. There is a critical point, beyond which the slow roll inflation phase disappears.

3. Discussion

3.1. On primordial density perturbation at the eternal inflation boundary

In single field inflation, when the inflaton field is in its eternal inflation boundary, the primordial perturbation $\delta\rho/\rho \sim 1$. The primordial density perturbation during Nflation can be calculated by using the formula of Sasaki and Stewart [22]. In slow roll approximation, $(\frac{\delta\rho}{\rho})^2 \sim \frac{m^2N^2\phi^4}{M_p^6}$ [6,23]. The motion of the eternal inflation boundary obeys Eq. (2). Thus substituting Eq. (2) into it and then cancelling the variable ϕ, we can obtain

$$\frac{\delta\rho}{\rho} \simeq \frac{1}{\sqrt{N}}, \tag{4}$$

where the factor with order one has been neglected, which hereafter will also be implemented. We can see that $\delta\rho/\rho$ is decreased with respect to the increase of N, and for each value of N, $\delta\rho/\rho$ is always less than one. This result is obviously different from that of single field. The reason leading to this result is, in single field inflation the eternal inflation boundary and the point that the density perturbation equals to one are same, however, the changes of both with N are different, one is $\sim 1/N^{3/4}$ and the other is $\sim 1/\sqrt{N}$. Intuitively, the eternal inflation means that the quantum fluctuations of fields lead to the production of many new regions with different energy densities, thus it seems that when we approach the eternal inflation boundary the density perturbation will be expected to near one. Thus in this sense our result looks like unintuitive. However, in fact what the eternal inflation phase means should be a phase in which the quantum fluctuation of field overwhelms its classical evolution, which is not certain to suggest that the density perturbation is about one.

Thus different from single field inflation, in which we are impossible to receive the information from the eternal inflation phase since in that time the black hole has swallowed us due to the primordial density perturbation with near one, it seems that when N is large, we may obtain some information from the eternal inflation phase, at least in principle we can obtain those from the boundary of eternal inflation phase. Beyond this boundary, the fields are walked randomly, thus the slow roll approximation is broken and the results based on the slow roll approximation are not robust any more. In principle, for the eternal inflation phase of Nflation we need to calculate the density perturbation in a new way to know how much it is actually, which, however, has been beyond our capability. The eternal inflation phase for single fields has been studied by using the stochastic approach [24].

3.2. On entropy bound

The entropy during Nflation can be approximately given by dS entropy $S \sim \frac{M_p^2}{H^2}$. Here we regard S as the entropy at the eternal inflation boundary. Thus we have

$$S \sim \frac{M_p^2}{H^2} \sim \frac{M_p^4}{Nm^2\phi^2} \sim \sqrt{N}\frac{M_p}{m}, \tag{5}$$

where Eq. (2) has been used. It is interesting to find that S is proportional to $\sqrt{N}$, which means the entropy increases with the number of field. Here the case is slightly similar to that of the entanglement entropy for a black hole, in which there seems be a dependence of the entanglement entropy on N, which conflicts the usual result of black hole entropy, since each of fields equally contributes to the entropy [17]. However, this problem may be solved by invoking the correct gravity cutoff $\Lambda \sim \frac{M_p}{\sqrt{N}}$ [16], as has been argued in Ref. [17]. In Eq. (5) if we replace M_p with a same gravity cutoff Λ, then we will obtain $S \sim \sqrt{N}\frac{\Lambda}{m} \sim \frac{M_p}{m}$, which is just the result for single field, i.e. $S \sim \frac{M_p}{m}$ at the eternal inflation boundary. Thus it seems that the argument in Ref. [17] is universal for the relevant issues involving N species.

It can be noticed that the efolding number $\mathcal{N} \sim \frac{N\phi^2}{M_p^2}$. For initial ϕ being in its eternal inflation boundary, where ϕ is given by Eq. (2), for fixed N, i.e. along the line paralleling the ϕ axis in Fig. 1, $\mathcal{N}$ obtained will be the total efolding number along corresponding line in slow roll inflation phase, hereafter called $\Delta\mathcal{N}$, see Fig. 1. Thus with Eq. (2), we can have $\Delta\mathcal{N} \sim \frac{M_p}{m\sqrt{N}}$. Then we substitute it into Eq. (5), and thus for the eternal inflation boundary, we have

$$N \cdot \Delta\mathcal{N} \simeq S, \tag{6}$$

which is a general entropy bound including N, and is also our main result. It means that below the eternal inflation boundary, we have the bound $N \cdot \Delta\mathcal{N} \lesssim S$. This result indicates that for fixed N, i.e. along the line paralleling the ϕ axis in Fig. 1, the total efolding number $\Delta\mathcal{N}$ of slow roll inflation phase is bounded by S, while for fixed $\Delta\mathcal{N}$, i.e. along the line paralleling the N axis in Fig. 1, the number N of fields is bounded by S, and at the eternal inflation boundary, the entropy bound is saturated.

There are two special cases, corresponding to the regions around red points in Fig. 1. For details, one is that for $N = 1$, i.e. single field, we have $\Delta\mathcal{N} \simeq S$ from Eq. (6), thus the result for single field is recovered [21]. Following [21] to large N, Eq. (6) can be actually also deduced. By making the derivatives of $\mathcal{N}$ and S with respect to the time, respectively, we can have

$$\frac{d\mathcal{N}}{dS} \simeq \frac{M_p^2}{m^2 S^2}, \tag{7}$$

where S is the function of ϕ, see the second equation in Eq. (5), and thus can be used to cancel ϕ. By integrating this equation along the line paralleling the ϕ axis in Fig. 1, where the lower limit is the eternal inflation boundary and the upper limit is the end value of slow roll inflation phase, and then applying approximation condition $\phi_e \ll \phi$, where ϕ and ϕ_e represent the values of eternal inflation boundary and the end of slow roll inflation, respectively, which actually implies that $S_e \gg S$ and thus $(S_e - S)/S_e \simeq 1$, we have

$$\Delta\mathcal{N} \simeq \left(\frac{\delta\rho}{\rho}\right)^2 S, \tag{8}$$

where $(\frac{\delta\rho}{\rho})^2 \sim \frac{M_p^2}{m^2 S^2}$ has been applied, which can be obtained since both $\frac{\delta\rho}{\rho}$ and S are the functions of ϕ. This result has been showed in Ref. [21] for single field, however, since Eq. (8) is independent on the number N of fields, thus it is still valid for N fields. For single field inflation, $\frac{\delta\rho}{\rho} \sim 1$ only at eternal inflation boundary, thus we always have $\Delta\mathcal{N} \lesssim S$ for slow roll phase, i.e. the total efolding number is bounded by the entropy, which is saturated at eternal inflation boundary. Note that Eq. (8) is an integral result in which $\delta\rho/\rho$ with the change of ϕ and thus S is considered, which is slightly different from that in Ref. [25]. Thus combining Eqs. (4) and (8), we can find Eq. (6) again. This also indicates the result of Eq. (4) is reliable.

The other is that for N being near its critical point, in which approximately we have $\Delta\mathcal{N} \simeq 1$, thus we can obtain $N \simeq S$, i.e. S is saturated by the number N of fields. This can also be seen by combining Eq. (3) for the critical point and Eq. (5), in which we can find $S \simeq N$ at the critical point.

Thus below the critical point, $N \lesssim S$. From Eq. (5), $S \sim \sqrt{N}\frac{M_p}{m} \gtrsim N$ can be obtained. This means $N \lesssim (\frac{M_p}{m})^2$. In Ref. [16], it was argued that M_p is renormalized in the presence of N fields at scale m so that $M_p^2 \gtrsim Nm^2$, in other words, $N > (\frac{M_p}{m})^2$ is inconsistent. Here, if $N > (\frac{M_p}{m})^2$, then combining it and Eq. (5), we will have $N > S$, i.e. the number N of fields is larger than the dS entropy of critical point. This is certainly impossible, since intuitively it may be thought that there is at least a freedom degree for each field, thus the total freedom degree of N fields system, i.e. the entropy, should be at least N, while dS entropy is the maximal entropy of a system. Thus we arrive at same conclusion with Ref. [16] from a different viewpoint. This again shows the consistence of our result.

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